# The Roles of Modelling, Simulation, Heuristic and Exact Optimization in Public Transport

13th Conference on Advanced Systems in Public Transport (CASPT 2015) Rotterdam, July 19 to July 23, 2015.

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Zuse Institute, Technische Universität and MATHEON Berlin, Germany



These are the slides of the presentation I have shown in Rotterdam on July 22.

However, I have deleted all slides, pictures and diagrams where I am not sure about the copyright. This is due to the fact that I have distributed slides before, which were put on the internet by someone else and which contained a piece of a map. A lawyer discovered that map (there are law firms specializing in this "business"), and I had to pay a fine of more than 600 Euros. That is why I have become somewhat cautious.

These slides are for your personal information only.

Martin Grötschel

Thanks to the organizers!

Where do I come from?

What drives me?

Where are the problems?

## TU Berlin, Mathematisches Institut



#### Zuse Institute (ZIB)



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## CO@Work 2015

**Combinatorial Optimization at Work** 

13.07.2015 **Marika Karbstein Finalist of the Tucker Prize of the Mathematical Optimization Society** 

#### MSO in Public Transport

28.09.2015 **CO@Work 2015 - Combinatorial Optimization at Work** 





**Neuausrichtung des Copernicus-Preises** : Mit dem Copernicus-Preis zeichnen die Deutsche Forschungsgemeinschaft (DFG) und die Stiftung für die polnische Wissenschaft (FNP) zusammen ... **2. International MATHEON Conference on Compressed Sensing and its Applications**: Welcome to the 2. International MATHEON Conference on Compressed Sensing and its Applications 2015. ...



3. BIMoS-Day: The Berlin International Graduate School in Model and Simulation based Research (BIMoS) devotes each BIMoS Day to introducing a broad scientific audience to a novel methodology with a wide range of ...



#### Aufgelesen

- · Mathe ist "männlich", gute Schulleistungen "weiblich" (Tagesspiegel)
- Spätbabylonische Liebe zur Mathematik (Spiegel online)
- Die Mathematik des Riechens (Bild der Wissenschaft)
- Mathematikerin Hélène Esnault (Tagesspiegel)
- Zahlenplattform Imaginary.org (Spiegel online)
- Rechenschwäche. (Berliner Zeitung)
- Die neue "Miss Germany" ist gekürt (Berliner Zeitung)
- Neue Methoden im Mathe-Unterricht (Arte TV)
- Die Optimierung der Welt (Tagesspiegel)
- Mathe-Prof als YouTube-Star (Spiegel Online)



Anmeld

#### Forschungscampus MODAL: BMBF and industry supported



# Why am I interested in public transport?

My fields of mathematical interest are:

optimization, discrete mathematics and operations research

Public transport and traffic planning are among the application areas offering the biggest challenges from any perspective:

- data acquisition (getting all data, keeping them up-to-date, processing the data,...)
- modelling the concepts, questions and tasks mathematically
- developing reasonable simulation environments
- creating and using appropriate theory
- employing the right/fitting solution methodology
- large scale online and real-time problems
- really huge instance sizes (big data before this became fashion)
- impact for the public and for industry

# Why am I interested in public transport?

Problems of the field are:

- Many public transport companies do not know exactly what they want: moving target problem
- Politicians interfere: How should the subsidies be used?
- Customers have different/varying objectives
- Software companies providing "solutions" often make gigantic claims and promisses
- We optimize the resources of the world
- We have the most powerful solution engines
   Marketing
- ...can make your solution even more optimal

#### The claims and buzzwords

XYZ's technology is based on sophisticated novel advanced mathematical algorithms combined with artificial intelligence and operations research techniques, developed together by mathematicians, computer scientists and transportation planning experts.

XYZ's unique technology is able to create fully optimal solutions within seconds taking the full set of constraints and preferences into account. Our most powerful solution engine is modular and integrated, robust and easily customizable providing the greatest possible flexibility... There is no planning problem we can't solve, no business operation we can't optimize.

That is what, in some way or the other, some vendors state. How can a customer react to such overblown marketing buzz words?

What do you mean by optimized? What methodology do you really use? Which algorithms do you employ? What optimality guarantee can you provide? Have you ever heard of *NP*-completeness?



#### We should be happy

- to really understand the problems in public transport
- to formulate (model) the desires/wishes/goals in a way such that everyone involved understands them
- to be able to acquire the necessary data
- to standardize/automatize the solution approach
- to solve the problems addressed a little better than before and
- that the solutions are accepted by the customers.

These are the honest statements of a mathematician, but, of course, this is not marketing language.

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- 10. The solution technologies (exact & heuristic)
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# Modelling?

Modelling: What is that?

#### Picasso: "Art is a lie that makes us realize truth"

Olga Chochlowa Picasso's first wife

The model of the mathematician

The model of the artist

A mathematical model is a lie that helps us see the truth.

# Picasso and his women



#### Francesca

Marie-Thérèse





### Confusion: There are many other ways of modelling

- computer models
- business model
- architectural models
- chemical models
- medical models

#### Modelling: What is that? car industry

# modell car car modell

Elvis' gold-leafed Cadillac (a gift from Priscilla).

1960 Series 75 Fleetwood Limousine

This vehicle is the ultimate in luxury and opulence. Costing \$100,000, it was customised by George Barris with gold plated interior gadgets such as a phone, shoe buffer, refrigerator, entertainment console with a ten record automatic changer RCA record player, swivel TV and tape deck. Forty coats of exterior paint made with pearl, diamond dust and oriental fish scales were used on the outside. Hubcaps, wheel covers, headlight rims and the front grille are gold plated in 24kt gold. Gold lame drapes were used to cover the back windows and to separate the front and back seats. Truly a vehicle fit for a King!

#### Modelling: What is that? architecture

# Massenmodell

Martin Grötschel

#### Modelling: What is that? chemistry

# Model Railway/Modelleisenbahn

#### Mathematical Modelling: What is that?

Beginning with observations

- ▷ of our environment
- ▷ a problem in practice of particular interest
- ▷ a physical, chemical or biological phenomenon
- an organizational/planning task

possibly with guiding/tailored experiments and including data acquisition:

the attempt of a formal representation via "mathematical concepts" (variables, equations, inequalities, objective functions, etc.), aiming at the utilization of mathematical theories and tools.

## Optimization problems that model reality (occasionally)

$$\max f(x) \text{ or } \min f(x)$$
$$g_i(x) = 0, \quad i = 1, 2, ..., k$$
$$h_j(x) \le 0, \quad j = 1, 2, ..., m$$
$$x \in \mathbb{R}^n \text{ (and } x \in S)$$

$$\min c^{T} x$$
$$Ax = a$$
$$Bx \le b$$
$$x \ge 0$$

$$\min c^{T} x$$

$$Ax = a$$

$$Bx \le b$$

$$x \ge 0$$

$$x \in \mathbb{Z}^{n}$$

$$(x \in \{0,1\}^{n})$$
(linear)

"general" (nonlinear) program NLP

linear program LP (linear) integer program IP, MIP

program = optimization problem

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#### Simulation

Simulation, Simulator or simulate are derived from the latin words *simulare* and *similis*.

They mean: pretend to be or the same sort.

## Simulation

", Computation" of several (close to reality) variants of a mathematical modell aimimg at:

- validation of the correctness of a model
- investigation of typical instances in the model framework, e.g., to avoid experiments or to test some functionality (crash-test)
- good predictions (weather)
- computation of reasonable solutions for the control of a system in practice (control of transport and logistics-systems)



Computation of instances varying several parameters

Parameters of a car crash test, e.g.: speed, material stiffness, various angles





Dynamik eines ' $\Delta$ '–hedge gegen einen Garantiefonds in verschiedenen Szenarien.





3D-reconstruction of a scull from a magneto-resonance tomografic investigation

#### **Tsunami Model Simulation**

# Simulation in traffic and transport

The **Nagel–Schreckenberg model** is a theoretical model for the simulation of freeway traffic. The model was developed in the early 1990s by the physicists Kai Nagel and Michael Schreckenberg. It is essentially a simple cellular automaton model for road traffic flow that can reproduce traffic jams, i.e., show a slow down in average car speed when the road is crowded (high density of cars).

#### Train Simulation Tools

#### SBB trains live – Die Schweizer Bahn als Simulation

Swisstrains.ch zeigt Ihnen fast alle Zuege in der Schweiz auf einer Karte. Wenn Sie einen Zug selektieren und clicken auf "follow", koennen Sie einen Zug folgen auf seine Reise… Bald verfuegbar (ende des Jahres): die iPhone-Version fuer Swisstrains.



#### Link: http://www.swisstrains.ch/

#### European "Bottleneck"



#### **MTO Simplon**



<u>www. swi sstrains. ch</u>

# MTO Simplon Netzdaten

- ca. 45.3 km Strecke (Brig-Domodossola II
- > 12 Bahnhöfe
- > 560 (2\*577=1154) OpenTrack Mikroknoten
- > 1831 OpenTrack Mikrokanten
- > 223 Signale
- > 8 Überleitstellen
- > 100 Weichen
- 6 Zuggattungen
- > 40 Laufwege
- > 230 Fahrstrassen (Blöcke)
- > makrotisiert zu 18 Stations und 40 Tracks



#### Infrastrukturdaten

#### Complex Traffic at the Simplon





Source: Wikipedia

#### Slalom route

ROLA trains traverse the tunnel on the "wrong" side

#### Crossing of trains

complex crossings of AUTO trains in Iselle

#### Conflicting routes

 complex routings in station area Domodossola and Brig



#### Detailed railway infrastucture data (OpenTrack)



Signals

- ▷ Switches
- Tracks (with max. speed, acceleration, gradient)
- Stations and Platforms


Generation of artifical nodes – pseudo stations





# Time Discretization Analysis

Time discretization dt/s	6	10	30	60
Number of trains	196	187	166	146
Cols in IP	504314	318303	114934	61966
Rows in IP	222096	142723	53311	29523
Solution time in Sek.	72774.55	12409.19	110.34	10.30

Adaptive discretization needed



### Theoretical Network Capacity (B., Schlechte, Swarat [2010])



- 180 trains for network small (without station routing and buffer times)
- 196 trains for network big with precise routing through stations (without buffer times)
- 175 trains for network big with precise routing through stations and buffer times

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# What is optimization?

### Optimization in the weak sense: To make things better!

This is the world of heuristics, intuition, trial and error, improvement efforts, etc.

We "optimized" in talks and publications usually does not mean that an optimal solution was found. It is often just an euphemism or advertisement.

### Optimization in the weak sense: To make things better!

The general standard example of mathematical optimization:

Given a set S and a function f from S to an ordered set T. Find a point x in S such that f(x) is as large or as small as possible (according to the ordering of T).

$$\max \text{ or } \min \left\{ f(x) \middle| x \in S \right\}$$

Usually, S is a subset of the n-dimensional vector space over the reals (in theory); in practice we can only compute with rational numbers, though.

And also usually, the ordered set T is the set of real or rational numbers.

# Typical optimization problems

$$\max f(x) \text{ or } \min f(x)$$
$$g_i(x) = 0, \quad i = 1, 2, ..., k$$
$$h_j(x) \le 0, \quad j = 1, 2, ..., m$$
$$x \in \mathbb{R}^n \text{ (and } x \in S)$$

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(linear)
integer

"general" (nonlinear) program NLP

linear program LP

J program IP, MIP

program = optimization problem

## **Complexity Theory**

Polynomial time solvability (fast): E.g., Dijkstra's algorithm for shortest paths

NP-completeness (difficult): E.g., integer programs

Practical efficiency: experimental mathematics

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# Kalyanmoy Deb: The great confusion

From his book: "Multi-objective optimization using evolutionary algorithms" (Wiley, 2001)

### Preface

Optimization is a procedure of finding and comparing feasible solutions until no better solution can be found. (Really?)

Classical optimization methods can at best find one solution in one simulation run, thereby making those methods inconvenient to solve multi-objective optimization problems. (???)

Evolutionary algorithms (EAs), on the other hand, can find multiple optimal solutions in one single simulation run due to their population-approach. (True?) Thus, EAs are ideal candidates for solving multi-objective optimization problems. (Very convincing!)

# Interplay

- Such uninformed and misleading statements are really a pity.
- They confuse everyone and just show that the persons making such statements have not really understood what they are doing.
- It is important to understand the roles of modelling, simulation and optimization, the differences between theses and what each of them contributes to the process of solving real problems. See the Simplon tunnel example.
- Each tool is indispensable, "best practice" requires the employment of all three techniques and a concept of managing the interplay.

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# Integer/combinatorial optimization models

- 1. travelling salesman problem (the prototype problem)
- 2. Chinese postman problem
- 3. shortest path and related graph problems
- 4. max-flow and min-cost flow problem
- 5. multicommodity-flow problem
- 6. set-packing, -partitioning, -covering
- 7. general integer and mixed-integer programs
- general routing
- location
- cut problems (max and min)
- scheduling
- further variants of flow problems
- dynamic programming
- etc.
- Complicating side constraints
- capacities, time windows, maintenance and resource constraints, regularity, periodicity, orientation, robustness, laws, labor rules...

# Typical optimization problems

$$\max f(x) \text{ or } \min f(x)$$

$$g_i(x) = 0, \quad i = 1, 2, ..., k$$

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"general"

(nonlinear)

program

NLP

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$$Ax = a$$
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LP

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# Berlin: Länderallee 11 to Garystr.35

A	Länderallee 11, Berlin				
В	Garystraße, Berlin				
_	Ziel hinzufügen - Optionen anzeigen				
		ROUTE BERECHNEN			
Ŧ	Vorgeschlagene Routen				
	A115 und Ausfah Hüttenweg	• Keine Verkehrsinformatio	ten nen		
	Clayallee • Be	ayallee 10,1 km, 16 Minute • Bei aktueller Verkehrslage: 19 Minute			
	A100	15,4 km, 17 Minuten aktueller Verkehrslage: 20 Minuten			

- Shortest path
- Fastest path
- Most ecological path
- Cheapest path
- Compromise path

Multicriteria optimization is difficult

# Three good paths: Which is best?

## How do I drive?

7,7 km 18 min



10 km 16 min

# Network, Line and Fare Planning (Potsdam)



### Multiobjective optimization Potsdam: Travel Time vs. Line Costs



Line Costs

### Scenario Toscana: Cost vs. Line Changes (VS-OPT) (35 lines, 585 trips, 233 462 arcs)



Monday Optimizing Transit Route Design for Equity Kelly Bartolaccini, Nicholas Lownes and Travis Waller

"...This paper explore the effects of designing transit routes for eight possible objective function formulations,..."

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Example: Routing automated guided vehicles in a container terminal in Hamburg Harbor

Cooperation with



Rolf Möhring's group

# Unit" of the service fleet:

# Yellow Angel

# gelber Engel



**Supereinsatz.** In Berlin und Brandenburg mussten die Gelben Engel letztes Jahr mehr als 240 000 Mal ausrücken, um Havaristen in der Hauptstadt und auf 1700 Autobahnkilometern wieder flottzumachen – ein Rekordeinsatz. Einen Rückgang von zehn Prozent bei den Pannen registrierten dagegen die Gelben Engel in Mecklenburg-Vorpommern. Bei insgesamt 72 389 Einsätzen schafften sie jedoch auch einen Rekord: In 84 Prozent der Fälle konnten die Autofahrer mit dem Wagen weiterfahren.

# Service vehicle planning at ADAC



# Watching/Controlling Public Transport

ITCS: Intermodal Transport Control System CAD/AVL: Computer Aided Dispatch/Automatic Vehicle Location AVLS: Automatic Vehicle Location System



## Traffic Control Center/Betriebsleitstelle

## Online and real-time optimization

# Online and real-time optimization



### Ist mein Zug pünktlich?

Ist mein Zug pünktlich oder hat er Verspätung? bahn.de und die mobilen Services der Deutschen Bahn informieren Sie aktuell und zuverlässig.

Suchen Sie einfach nach einem Bahnhof, nach den Zügen Ihrer Reisestrecke oder direkt nach der gebuchten Reiseverbindung auf Ihrer Fahrkarte.

Selbstverständlich finden Sie die aktuellen Informationen auch in der normalen Reiseauskunft ab zwei Stunden vor Abfahrt.

### Aktuelle Infos auf bahn.de, m.bahn.de oder in der DB Navigator App

### Aktuelle Meldungen

Hier informieren wir Sie über kurzfristige Behinderungen im Zugverkehr.

→ Zu den Verkehrsmeldungen

## Online and real-time optimization

### A big unsolved issue in public transport:

Almost everywhere, experienced dispatchers, following given rules, reschedule in case of sudden accidents, breakdowns, emergencies.

Adjustment of timetables after catastrophes (landslides, floods, power breakdown, IT system failures,...) is necessary, but a big challenge with respect to data acquisitions and optimization methodology.

# Not really real-time

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### **Definition?**

Several talks at this conference with intuitively similar but mathematically not identical definitions of robustness.

Robust optimization (has become a somewhat standardized term in the optimization community)

## Observations and Conclusions from a Business Perspective

### Stochastic model and simulation make it possible to judge robustness KPIs:

- Simulation shows that robustness of plans used in practice is non-optimal.
- ▶ KPI-based approaches can in principle yield satisfactory results in creating robust plans.
- But: Stochastic robustness measures can still improve on this and avoid issues in defining the trade-off between virtual cost for robustness KPIs and other optimization objectives (in particular: real cost).

### Robustness must consider both resources aircraft and crew:

- Aircraft- and crew-propagated delay have to be considered and optimized together to achieve practically meaningful results.
- ▶ In practical applications, transfer passengers may need to be considered as well.

#### **Robustness should be included in the plan way before the tail assignment phase:**

- Minimization of propagated delay in aircraft rotations and following preplanned rotations seem to be partially contradictory objectives 
  Crew-rotations should already be based on robust aircraft rotations.
- Retiming can yield considerable improvements in robustness.

Lufthansa Systems
- Tail Assignment = construction of rotations for individual aircraft.
- Constraints
  - Airport curfews
  - Minimal turn time
  - Preassigned activities
  - Maintenance constraints
- Stability
  - Small changes in input data can significantly increase the cost of the schedule, or even make it infeasible.

- Robustness = insensitivity of a solution to changes in input data
- ▶ Aim: Find a compromise between price and stability.
- For discrete problems it is a priori not clear what robustness is, because changes in the input data can change the structure of the problem.
- Possible robustness criteria for tail assignment
  - Decrease the number of propagated delays
  - Increase punctuality of flights
  - Increase stability of the solution with respect to disturbances
  - Decrease costs needed to repair disturbed plan

▶ ...



#### Tail Assignment

- Goal: Decrease impact of delays.
- Primary disruptions: genuine disruptions, unavoidable.
- Secondary disruptions: propagated disruptions.



- Additional buffer flight between flights can prevent propagation of delay to the consecutive flight.
- 30% of all delays are secondary.

#### Flight Duration and Delay Propagation



Gate phase

Represents ground delay at departure.

Model: aircraft waits until scheduled departure (but at least minimum ground time), then we decide, if flight is delayed and how long.

Random variables:

Probability of gate delay.

- Specified for every airport and day hour.
- Length of gate delay.
  - Log-normal distribution, tail follows
     Power-law distribution.
  - Parameters of distributions are same for all airports.



# Flight phase

Random variable represents gate-to-gate time.

Follows Log-Logistic distribution.

Parameters are estimated for every flight based on scheduled flight duration.





#### Variables

Path und config usage (request i uses path p, track j uses config q)

#### Constraints

Path and config choice

Path-config-coupling (track capacity)

#### **Objective Function**

Maximize proceedings and robustness

# **Computational Results**

# Scenarios for one day, single fleet, no maintenance rules. (18-24 aircraft, 100-170 legs)

- ▶ KPI: Bonus for every ground time buffer minute.
  - Threshold for maximal ground time buffer bonus minutes (15 minutes).
- ▶ EPD: Minimize expected aircraft delay propagation.

			improvement
Scenario	KPI	EPD	(%)
SC1	35,11	33,56	4,42
SC2	34,28	33,98	0,89
SC3	19,11	18,56	2,87
SC4	39,10	37,26	4,71
SC5	26,12	25,63	1,89
SC6	41,81	41,07	1,77
SC7	20,26	19,74	2,57
SC8	19,80	19,02	3,98

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$$\max f(x) \text{ or } \min f(x)$$

$$g_i(x) = 0, \quad i = 1, 2, ..., p$$

$$h_j(x) \le 0, \quad j = 1, 2, ..., q$$

$$x_i \in \mathbb{Z} \quad for \text{ some } i$$

$$x_j \in \{0, 1\} \quad for \text{ some } j$$

$$x_k \in \mathbb{R}^n \quad for \text{ some } k$$

#### Plus:

Stochastic aspects Robust optimization Real time/online optimization Several objectives

More general: mixed-integer nonlinear program MINLP

# Theoretical vs. practical efficiency

There is no really hard evidence:

NP-hard problems seem to be hard in practice,

- but: many practical instances can be solved in reasonable time, even if they are large scale
  - but: a small variation of a well-solved hard problem may make it very hard in practice (e.g., time windows)

This particularly applies to public transport.

Problems solvable in polynomial time may be difficult or even unsolvable in practice.

How can you know? Testing, experiments, testing,...

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A joint view is necessary

#### Modelling is not easy! Modelling and solution approach can't be separated, in general

What is an appropriate model? What is the "right" LP relaxation? How exact need answers to be?

#### Problem classes



Mixed Integer Programming

$$\min c^{T} x$$

$$Ax = a$$

$$Bx \le b$$

$$x \ge 0$$

$$\operatorname{some} x_{j} \in \mathbb{Z}$$

$$\operatorname{some} x_{k} \in \{0, 1\}$$

(linear) 0/1- or mixed-integer program IP, MIP the "key problem" in traffic and transport

# Modelling and solution technologies interact

Issues such as:

- multiperiod planning
- coarse to fine modelling
- multi-scale aspects (e.g. planning in seconds and months within one problem instance)

have a significant impact on the solution methodology.

Techniques, e.g.:

- decomposition (Dantzig-Wolfe, Benders,...)
- hierarchical (master-slave) approaches
- Lagrangian relaxations
- dynamic programming

# Heuristics: A Survey

- Greedy Algorithms
- Exchange & Insertion Algorithms
- Neighborhood/Local Search
- Variable Neighborhood Search, Iterated Local Search
- Random sampling
- Simulated Annealing
- Taboo search
- Great Deluge Algorithms
- Simulated Tunneling
- Neural Networks
- Scatter Search
- Greedy Randomized Adaptive Search Procedures

#### Heuristics: A Survey

- Genetic, Evolutionary, and similar Methods
- DNA-Technology
- Ant and Swarm Systems
- (Multi-) Agents
- Population Heuristics
- Memetic Algorithms (Meme are the "missing links" gens and mind)
- Fuzzy Genetics-Based Machine Learning
- Fast and Frugal Method (Psychology)
- Method of Devine Intuition (Psychologist Thorndike)
- ....
- We have great researchers who implemented a unique superfast solution technology

There is (sometimes) a marketing battle going on with unrealistic, or even ideological, claims about the quality of heuristics – just to catch attention

Linguistic Overkill:

Simulated hybrid meta GA-based neural evolutionary fuzzy variable adaptive search parallel DNA-driven multi-ant-agent method with devine swarm taboo intuition

#### An Unfortunate Development

There is (sometimes) a marketing battle going on with unrealistic, or even ideological, claims about the quality of heuristics – just to catch attention

Linguistic Overkill:

# Vodoo Approach

Heuristics are very important, of course. It would be good to make honest statements about them.

# Algorithms for the Solution of LPs

- 1. Fourier-Motzkin Elimination
- 2. The Primal Simplex Method
- 3. The Dual Simplex Method
- 4. The Ellipsoid Method
- 5. Interior-Point/Barrier Methods
- 6. Double Description, Over Relaxation
- 7. Lagrangian Relaxation, Coordinate Ascent/Gradient/Subgradient/Bundle Methods
- 8. Decomposition Techniques

# Algorithms for the Solution of LPs

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#### Algorithms for the Solution of MIPs

- 1. Heuristics
- 2. Branch&Bound
- 3. Cutting Planes
- 4. Flow Techniques
- 5. Column Generation
- 6. Benders Decomposition
- 7. Constraint Programming
- 8. Domain Decomposition
- 9. Conflict Analysis
- 10. Other Methods

#### MIP Solver Techniques



Branch & Bound





Conflict Analysis



# A Branching Tree

#### sw24978 Branching Tree

Computation Carried out in Parallel at Georgia Tech, Princeton, Rice

Applegate Bixby Chvátal Cook

#### tree copied from

www1. ctt. dtu. dk/ROUTE2003/
presentations/cook. pdf



# Cutting plane technique for IP and MIP

Feasible integer solutions

Objective function

Convex hull

LP-based relaxation

Cutting planes



#### Bus circulation: Flow through a bus line network



Multicommodity flow with minimum cost







#### **Urban Scenarios**

	BVG	HHA	VHH
depots	10	14	10
vehicle types	44	40	19
timetabled trips	25 000	16 000	5 500
number of variables	70 000 000	15 100 000	10 000 000
cpu mins	200	50	28

Column Generation can be viewed as a procedure dual to the cutting plane method. The basic principle:

- Select a small number of variables and solve the linear program (or LP relaxation) using only these.
- 2. Find an unused variable (or several) which, if included, would (most) improve the objective value, or determine that there is none, i.e., the linear program has been solved: stop.

Solve the **column generation subproblem (Pricing)**.

Model this as an optimization problem, works best if this is an easy IP (shortest path, dynamic program, etc.)

3. Include the variable(s) in the linear program, re-solve it, and go to step 2.

#### Successful Applications: Examples

# Cutting stock problems (Gilmore, Gomory) **1960**

- Air crew scheduling
- Aircraft fleeting and routing
- Crew rostering
- Vehicle routing
- **Driver** assignment
- **Global shipping**
- Multi-item lot-sizing
- Telecommunications network design
- Cancer radiation treatment using IMRT

#### **Column Generation**

This method is particularly important if lots of rules have to be satisfied that are not easy to model, blow up the IP enormously and are subject to frequent changes, such as the work rules for drivers in public transport or pilots and crew in the airline business. (I can give a full hour lecture on "break rules".)

The "difficult side constraints" are treated in the column generation subroutine(s).

# Telebus (transportation of disabled persons)



#### Graph Theoretical Model, IP: Set Partitioning Model



## Savings May 94 – May 96

Martin Grötschel

#### Historie: Telebus-Projekt

Ziele

- Organisatorische Verbesserungen
- Mathematische Fahrzeugeinsatzplanung



#### Ergebnisse

- Serviceverbesserung
- Kostenreduktion
- Vereinfachung der Arbeitsabläufe
- Telebus-Computersystem

#### Computational Results for a Duty Scheduling Set Partitioning Model

#### Duty Scheduling Problem Ivu41:

- 870 500 col
- 3 570 rows
- 10.5 non-zeroes per col

Coordinate Ascent: Fast, low quality Subgradient: (Theoretical) Convergence Volume: Primal approximation Bundle+AS: Conv. + primal approx. Dual Simplex: Primal+dual optimal Barrier: Primal+dual optimal


#### Integrated Bus and Driver Scheduling: Model Structure



### Integrated vehicle and driver optimization

Despite of the high productivity the automated planning, the integrated optimization brings further improvements

#### **Example optimization**

Slide of DB













#### The role of heuristics in MIP in MIP solving

Primal MIP heuristics, examples:

- Rounding Heuristics
- Diving Heuristics
- Large Neighborhood Search
- Local Branching
- RINS
- Crossover
- DINS
- RENS (Relaxation Enforced Neighborhood Search)
- Feasibility Pump(s)
- Undercover

### special "simple" combinatorial optimization problems

Finding a

. . .

- minimum spanning tree
- shortest path
- maximum matching
- Chinese Postman tour
- maximal flow through a network
- cost-minimal flow

solvable in polynomial time by special purpose algorithms

### Special "hard" combinatorial optimization problems

- travelling salesman problem
- location und routing
- set-packing, partitioning, -covering
- max-cut
- linear ordering
- scheduling (with a few exceptions)
- node and edge colouring
- ...

NP-hard (in the sense of complexity theory)

The most successful solution techniques employ linear programming.

Lessons learned from these have now entered the general tools developed for general MIP solving.

All modern efficient LP and MIP solvers are, in addition to implementing the mathematics of the codes correctly, a huge conglomerate of heuristics aiming at speeding up evry step of the methods to achieve best performance in practice.

This is tested on large collections of instances from customers.

### Independent Testing: http://plato.la.asu.edu/



#### Benchmarks for Optimization Software

#### by Hans Mittelmann (mittelmann at asu.edu)

Note that on top of the benchmarks a link to logfiles is given!

#### **COMBINATORIAL OPTIMIZATION**

Concorde-TSP with different LP solvers (7-2-2015)

#### LINEAR PROGRAMMING

- Benchmark of Simplex LP solvers (7-3-2015)
- Benchmark of parallel LP solvers (7-9-2015)
- Parallel Barrier Solvers on Large LP/QP problems (6-29-2015)
- Large Network-LP Benchmark (commercial vs free) (6-28-2015)

#### MIXED INTEGER LINEAR PROGRAMMING

- MILP Benchmark MIPLIB2010 (7-7-2015)
- The EASY MIPLIB Instances (7-8-2015) (MIPLIB2010)
- MILP cases that are slightly pathological (7-6-2015)
- Feasibility Benchmark (6-29-2015) (MIPLIB2010)
- Infeasibility Detection for MILP (7-5-2015) (MIPLIB2010)

#### SEMIDEFINITE/SQL PROGRAMMING

- SQL problems from the 7th DIMACS Challenge (8-8-2002)
- Several SDP codes on sparse and other SDP problems (12-14-2014)
- MISOCP and large SOCP Benchmark (7-11-2015)

#### NONLINEAR PROGRAMMING

AMPL-NLP Benchmark, IPOPT, KNITRO, LOQO, PENNLP, SNOPT, WORHP, XPRESS & CONOPT (6-29-2015)

#### **MIXED INTEGER QPs and QCPs**

MIQ(C)P Benchmark (7-2-2015)

#### MIXED INTEGER NONLINEAR PROGRAMMING

MINLP Benchmark (1-9-2014)

#### PROBLEMS WITH EQUILIBRIUM CONSTRAINTS

MPEC Benchmark (6-1-2015)

#### Martin Grötschel

#### Independent Testing: http://plato.la.asu.edu/





- Parallel Barrier Solvers on Large LP/QP problems (6-29-2015)
- Large Network-LP Benchmark (commercial vs free) (6-28-2015)
- MPEC Benchmark (6-1-2015)

#### Courtesy Bob Bixby

## MIP Speedups 1991 – 2008



**MIP: Computational Progress** 

# Gurobi MIP Speedups 2009-2014

- Gurobi 1.0 and CPLEX 11.0 roughly equivalent
- Gurobi version-to-version improvements
  Gurobi 1.0 -> 6.0: 29.4X

Courtesy Bob Bixby

- Overall improvement: 1990 to 2014
  Algorithms: 870,000x
  - Machines: 6,500x
  - NET: Algorithm x Machine 5,600,000,000x
    (180 years / 5.6B ~= 1 second)

#### What is the SCIP Optimization Suite?

The SCIP Optimization Suite is a toolbox for generating and solving mixed integer programs. It consists of the following parts:

- SCIP mixed integer (linear and nonlinear) programming solver and constraint programming framework
- SoPlex linear programming solver
- ZIMPL mathematical programming language
- UG parallel framework for mixed integer (linear and nonlinear) programs
- GCG generic branch-cut-and-price solver

The user can easily generate linear programs and mixed integer programs with the modeling language ZIMPL. The resulting model can directly be loaded into SCIP and solved. In the solution process SCIP may use SoPlex as underlying LP solver.

Since all five components are available in source code and free for academic use, they are an ideal tool for academic research purposes and for teaching mixed integer programming.

Download the SCIP Optimization Suite (Linux). Previous releases and versions for different platforms are available here.

### About

SCIP is currently one of the fastest non-commercial solvers for mixed integer programming (MIP) and mixed integer nonlinear programming (MINLP). It is also a framework for constraint integer programming and branch-cut-and-price. It allows for total control of the solution process and the access of detailed information down to the guts of the solver.



MIP solver benchmark (1 thread): Shifted geometric mean of results taken from the homepage of Hans Mittelmann (16/Jan/2015). Unsolved or failed instances are accounted for with the time limit of 2 hours.

### Download from http@scip.zib.de

SCIP users worldwide



#### Contents

- 1. What is modelling?
- 2. What is simulation?
- 3. What is optimization?
- 4. What is the difference?Is there an interplay?
- The best known/most used mathematical models in public transport
- 6. Multicriteria optimization
- 7. Online optimization
- 8. Robustness
- 9. Theoretical and practical complexity

- 10. The solution technologies (exact & heuristic)
- 11. Data acquisition
- 12. The "big" public transport picture
- 13. Further examples

We have all data, of course!

Big data:

a huge problem, a huge challenge and a huge chance!

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### Planning Process in Public Transport



Phase:	Planning	Scheduling	Dispatching
Horizon:	Long Term	Medium term Timetable Period	(very) Short term Day of Operation online planning
Objective:	Service Level	Cost Reduction	Get it done
Steps:	Network Design Line Planning Timetabling	Vehicle Scheduling Duty Scheduling Duty Rostering	Crew Assignment Delay Management Failure Management



done well

industry in general not ready



Santiago de Chile 2007

### What is a good public transit network?

	Santiago	Berlin	
Area	1 400 km²	900 km <sup>2</sup>	
Population	5.8 Mio.	3.4 Mio.	
commuter		164 stations	
train		329 km tracks	
		15 lines	
metro	52 stations	170 stations	
	45 km tracks	144 km tracks	
	3 (+3) lines	9 lines	
	since 1975		
tram		377 stations	
		188 km tracks	
		21 lines	
bus	7 000 buses	1 300 buses	
	3 000 companies		



#### Berlin Light Rail Network





### Potsdam Origin-Destination Matrix





#### Line Planning

Find a cost minimal set of lines and associated frequencies, s.t. a given set of travel demands can be transported in minimal time.

Prototype problem for line planning:

#### Path Connectivity

Find a set of paths to connect a (sub)set of nodes in a graph.

Steiner connectivity problem





# Line Planning: Model Comparison

	(CG) <sup>1</sup>	(DC) <sup>2</sup>	(BD) <sup>3</sup>
capacity constr.	$\mathcal{O}( \mathcal{L}  A )$	$\mathcal{O}( D  A )$	A
pass. vars	assoc. with lines	inde	ependent of lines
optimality gap <sup>4</sup> $\Delta$ direct traveler <sup>4,6</sup>	inf <sup>5</sup> 0.0%	0.161% 0.006%	0.068% 19.26%

- <sup>1</sup> (CG) Change-and-go model of Schöbel and Scholl [SS2005] (complete and detailed treatment of transfers)
- <sup>2</sup> (DC) Direct connection model of Borndörfer and Karbstein [BK2012,2013] (distinguish 0 and at least 1 transfer; dcmetric inequalities)
- <sup>3</sup> (BD) basic dynamic model of Borndörfer, Grötschel, Pfetsch [BGP2005] (tractable line planning model, rudimental treatment of transfers)
- <sup>4</sup> geometric mean on 13 instances after 10 hours of computation
- <sup>5</sup> root LP for 7 of 13 instances could not be solved within 10 hours
- <sup>6</sup> difference of direct travelers predicted by model with exact direct travelers in system optimum



### Bus: ViP vs. ZIB



Graphic:  $\operatorname{VISUM}$ , ptv AG

### Passenger Flow





### Potsdam – Lineplan 2010



### What would we like to have?

Optimize everything in an integrated fashion, examples:

- Network design
- Line (and frequency) planning
- Timetable construction
- Vehicle circulation
- Crew assignment
- Crew rostering
- Crew dispatching
- Fare planning

Dream not to become true in my lifetime

And find quality guarantees, i.e., upper and lower bounds on the optimum value of the problem instance considered.

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### Some Traffic and Transport Projects



### Public Transport Projects (ZIB and MATHEON)

Busses (Berlin and elsewhere) Telebus (Transportation of disabled persons) **Bus Circulation** Bus Driver Scheduling Integrated Vehicle and Driver Scheduling Timetable Exchange Subways and Light Railways Subway Time Tabling Vehicle Scheduling

Infrastructure Planning Line Planing Network Planning (Potsdam) Fare Planing

#### Airlines

Airline Crew Scheduling/Rostering Tail Assignment: Robustness Free Flight Planning

Railways Railway Track Allocation ICE Circulation

Spin-Offs : LBW, Intranetz

#### Contents

#### 13. Further examples

- a. Bus circulation
- b. Train/ICE circulation
- c. Driver scheduling
- d. Crew scheduling
- e. Tail assignment
- f. Planning the public transport system of a city
- g. Generating competition, Trassenbörse
- h. Tail assignment
- i. Free flight

- Data
- Old habits, very conservative environment
- Rule based versus objective/goal oriented decision making
- Psychology of decision making
- Users do not understand the mathematics used.
- Users are reluctant to employ mathematics because interfacing with optimization software needs knowledge and education.
- Large number of failed projects
- Big skepticism or overly ambitious projects
- But things have significantly improved in the last years

### The Roles of Modelling, Simulation, Heuristic and Exact Optimization in Public Transport

13th Conference on Advanced Systems in Public Transport (CASPT 2015) Rotterdam, July 19 to July 23, 2015.

# The END Thanks for your patience

