Timetabling and Passenger Routing in Public Transport

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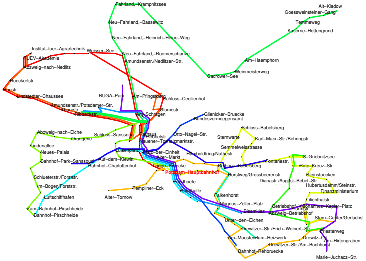
MATHEON - Network design & passenger behaviour in public transport

Routing Methods in Network Optimization

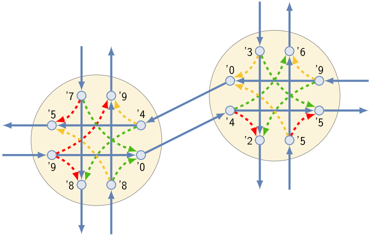
Line planning: simple flow models

- up to 10% less cost/travel time by integrating passenger routing [BN2008]
- 2-22% less travel time by direct connection routing [BK2012,K2013,BK2014]



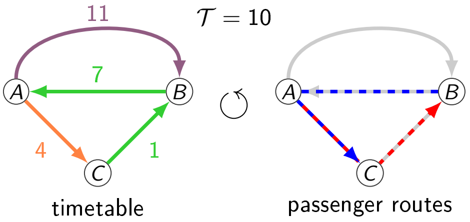
Timetabling: First approaches

- up to 30% less waiting time by integrating passenger routing [Lübbe2009]
- 2-15% less travel time by integrating passenger routing for aperiodic timetabling [Schmidt2012,Anhalt2012]



Lower Bound Routing vs. Shortest Path Routing

▷ lower bound routing



- demand:
  1 passenger from A to B
  1 passenger from B to C
- routing (right) is optimal for timetable (left) and vice versa

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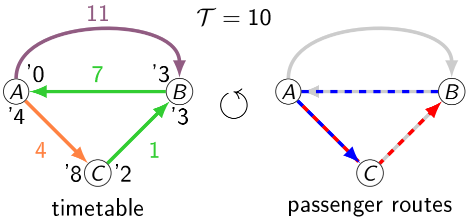

$\tau_{\text{total}} = 15 + 9 = 24$

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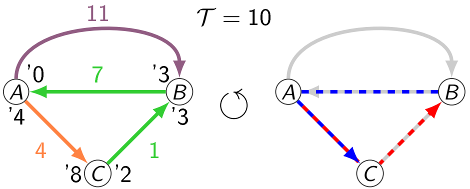
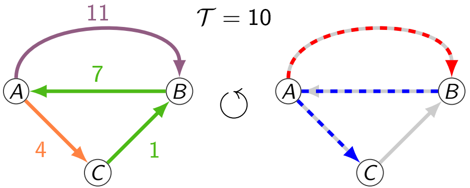


timetable

passenger routes

$\tau_{\text{total}} = 15 + 9 = 24$

▷ shortest path routing



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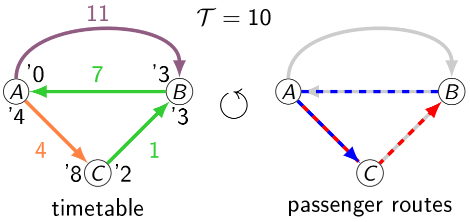

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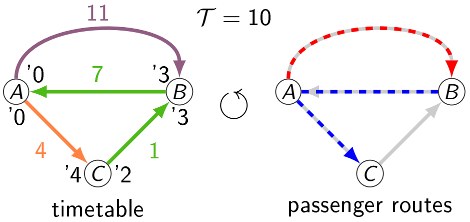

$\tau_{\text{total}} = 11 + 11 = 22$

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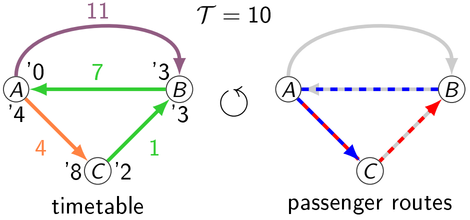
▷ lower bound routing



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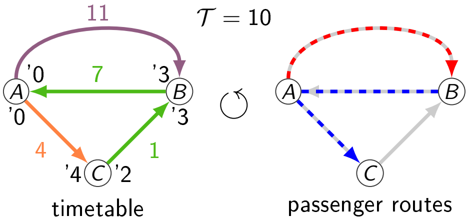

$\tau_{\text{total}} = 11 + 11 = 22$

- demand:
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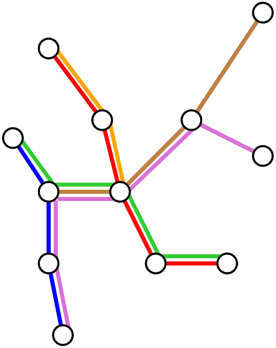
  1 passenger from B to C
- routing (right) is optimal for timetable (left) and vice versa

→ Iterated approach can be suboptimal.

Content

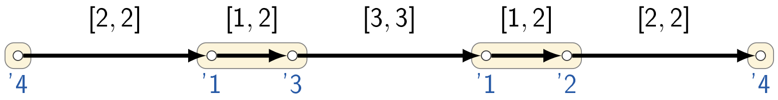
Periodic Timetabling with Passenger Routing



- Given:
  - transportation network
  - period time $\mathcal{T}$
  - lower and upper time bounds
  - passenger demand
- Wanted:
  - periodic arrival and departure times of lines at stations
- Objectives:
  - total travel time, total transfer time
  - (number of used vehicles)

Periodic Event Scheduling Problem (PESP)

$\mathcal{T} = 5$



[Serafini and Ukovich, 1989]:

Given: *Event-activity network* $N = (V, A)$, *period time* $\mathcal{T} \in \mathbb{N}$, lower and upper time bounds $\ell, u \in \mathbb{Q}^A$.

Wanted: Timing $\pi \in [0, \mathcal{T})^V$ of each event s.t. for all $a = (v, w) \in A$:

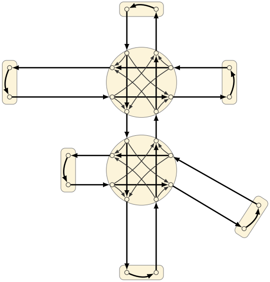
$$(\pi_w - \pi_v - \ell_a) \bmod \mathcal{T} \leq u_a - \ell_a.$$

π is called *periodic potential*.

Passenger Routing

Passenger demand:

- OD-matrix $d_{st} \in \mathbb{Q}^{V \times V}$
- OD-pairs $\mathcal{D} = \{(s,t) \in V \times V : d_{st} > 0\}$
- Passenger paths $\mathcal{P}$



$$\min \sum_{(s,t)\in\mathcal{D}} \sum_{p\in\mathcal{P}_{st}} \sum_{a\in p} d_{st}\, \tau_a\, y_p$$

$$\sum_{p\in\mathcal{P}_{st}} y_p = 1 \qquad \forall\, (s,t) \in \mathcal{D}$$

$$y_p \geq 0 \qquad \forall\, p \in \mathcal{P}$$

$\rightarrow$ Minimize total passenger travel time.

Passenger Routing & Timetabling

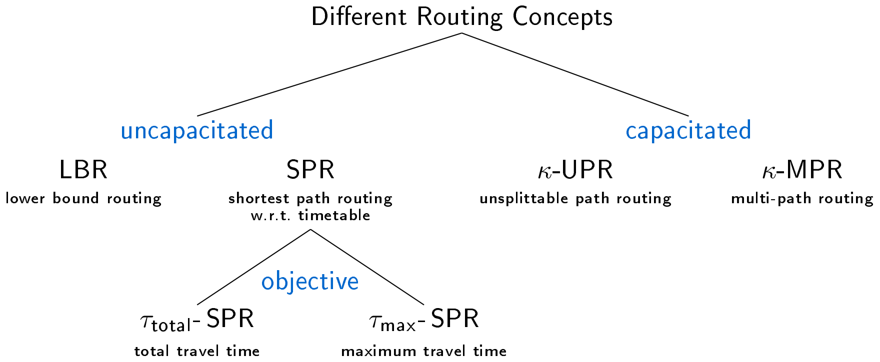
(PTPR)

$$
\begin{aligned}
\min \quad & \sum_{(s,t)\in\mathcal{D}} \sum_{p\in\mathcal{P}_{st}} \sum_{a\in p} d_{st}\, \tau_a\, y_p \\
(\pi_w - \pi_v - \ell_a) \bmod \mathcal{T} &\le u_a - \ell_a && \forall\, a=(v,w)\in A \\
(\pi_w - \pi_v - \ell_a) \bmod \mathcal{T} + \ell_a &= \tau_a && \forall\, a=(v,w)\in A \\
\pi_v &\ge 0 && \forall\, v \in V \\
\tau_a &\ge 0 && \forall\, a \in A \\
\sum_{p\in\mathcal{P}_{st}} y_p &= 1 && \forall\, (s,t)\in\mathcal{D} \\
y_p &\ge 0 && \forall\, p \in \mathcal{P}.
\end{aligned}
$$

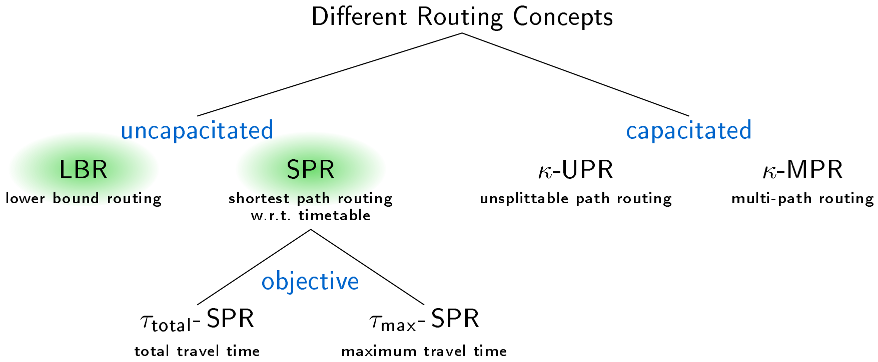
▷ Linearization:
periodic offset variables $p \in \mathbb{Z}^A$ and arc flow variables $w \in \{0,1\}^{\mathcal{D}\times A}$

Content

Routing Concepts



Routing Concepts



Lower Bound Routing vs. Shortest Path Routing

Restrict passenger paths $\mathcal{P}' \subseteq \mathcal{P}$

- lower bound routing (LBR): $\mathcal{P}'$ shortest path w.r.t. ℓ

  $\rightarrow$ fixed routing

- shortest path routing (SPR): $\mathcal{P}' = \mathcal{P}$

  $\rightarrow$ free routing

Lower Bound Routing vs. Shortest Path Routing

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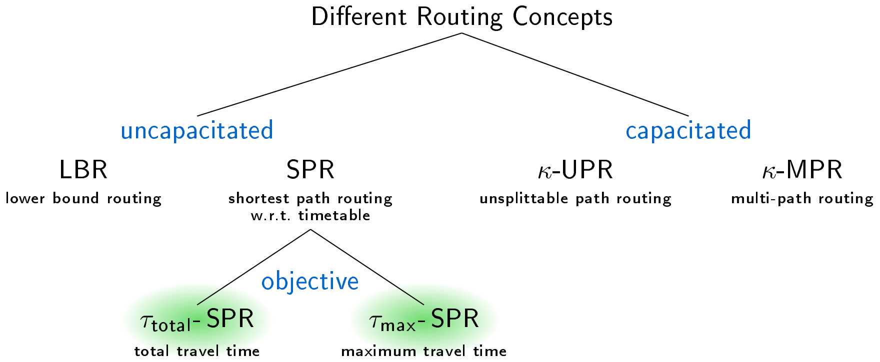
Proposition [BHK2015]:

$$\operatorname{gap}(\mathrm{LBR}, \mathrm{SPR}) := \sup_I \frac{v(\mathrm{LBR}; I)}{v(\mathrm{SPR}; I)} = \infty$$

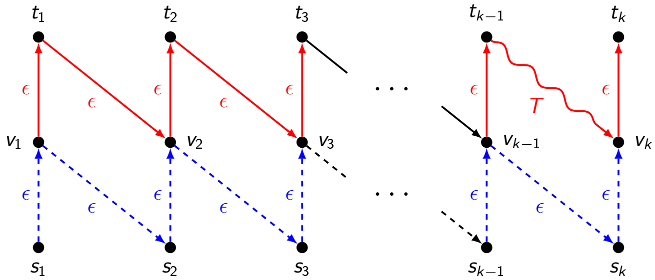
Routing Concepts



Maximum Travel Time vs. Total Travel Time

- τ_{total}- SPR: minimize total travel time
- $\tau_{\max}$- SPR: minimize maximum travel time

$\epsilon = \frac{\mathcal{T}+1}{k}$



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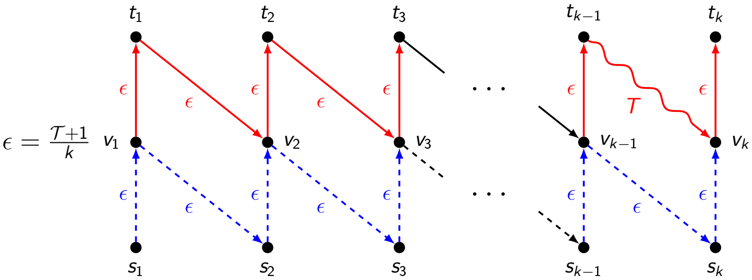

Lemma [BHK2015]:

$$\frac{\tau^{\text{total}}(\tau_{\max}\text{- SPR}; I)}{\tau^{\text{total}}(\tau_{\text{total}}\text{- SPR}; I)} \leq |\mathcal{D}| \quad \forall\, I$$

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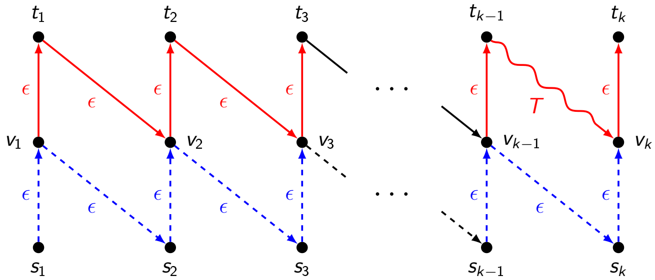

Proposition [BHK2015]:

$$\text{gap}(\tau_{\text{max}}\text{- SPR}, \tau_{\text{total}}\text{- SPR}) := \sup_I \frac{\tau^{\text{total}}(\tau_{\text{max}}\text{- SPR}; I)}{\tau^{\text{total}}(\tau_{\text{total}}\text{- SPR}; I)} = \infty$$

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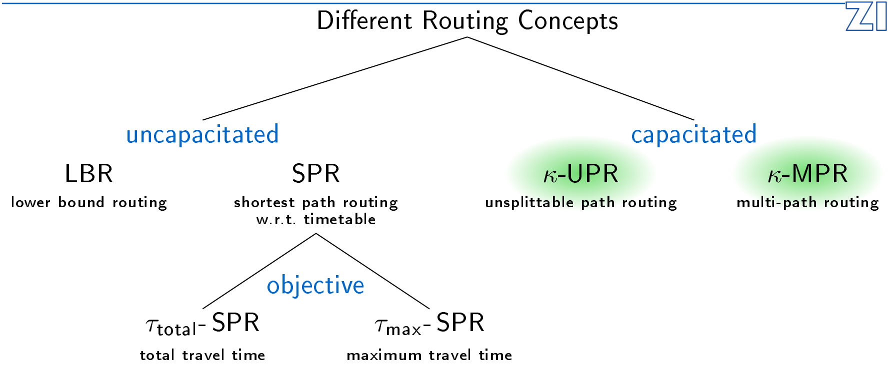
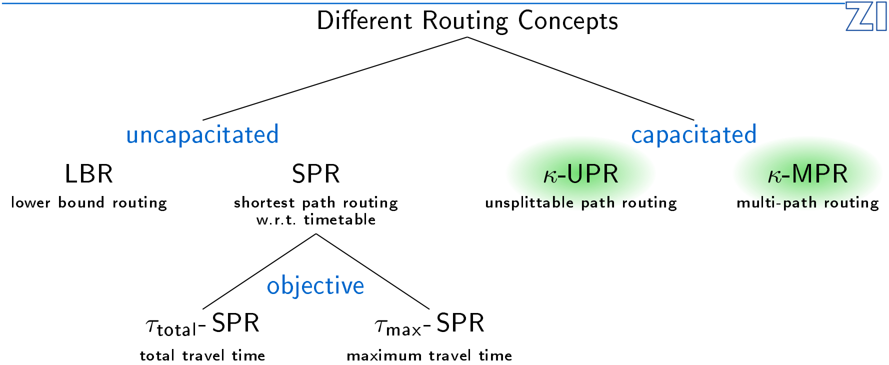
Routing Concepts



Routing Concepts

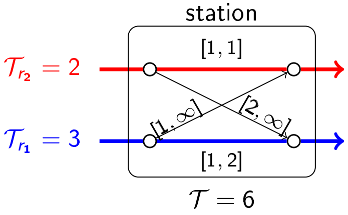


Proposition [BHK2015]:

$$\operatorname{gap}(\kappa\text{-UPR}, \kappa\text{-MPR}) = \infty.$$

Content

Time-Expanded Passenger Routing Network



Similar to network used by [Kinder, 2008] and [Müller-Hannemann et al., 2007]

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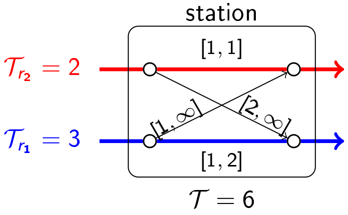

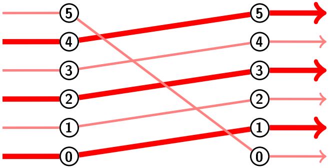

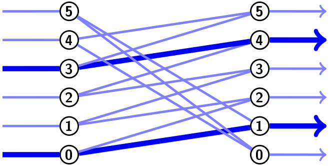

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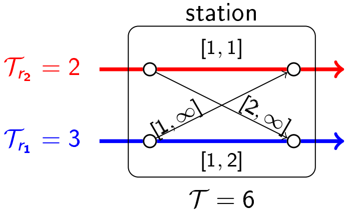

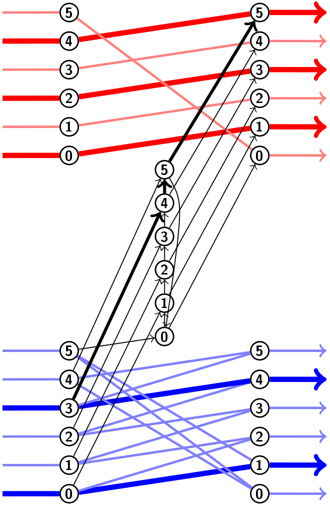

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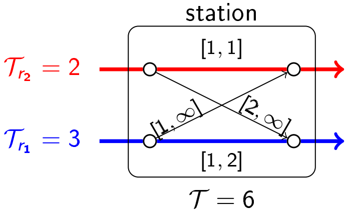

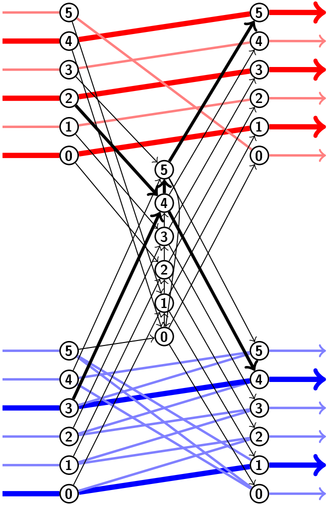

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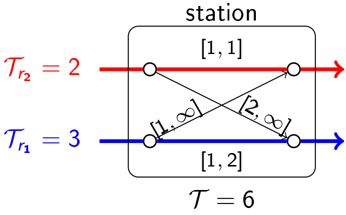
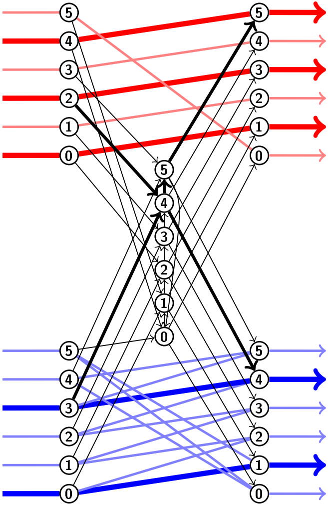
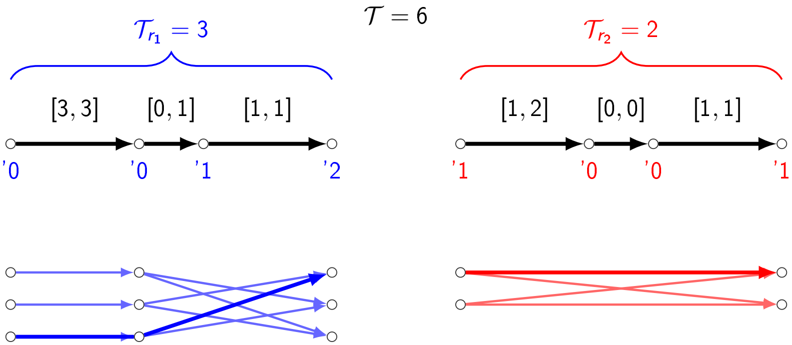




Lemma: $O(\mathcal{T})$ transfer arcs.

Similar to network used by [Kinder, 2008] and [Müller-Hannemann et al., 2007]

Time-Expanded Line Routing Network



Time-Expanded Integrated Model

$$(\mathrm{PTPR}^{\mathcal{T}}) \quad \min \sum_{(s,t)\in\mathcal{D}} \sum_{p\in\mathcal{P}_{st}} \sum_{a\in p} d_{st}\, \tau_a\, y_p$$

linear objective

$$\begin{aligned}
\sum_{\alpha\in\mathcal{A}^r_{\mathcal{T}}(b^r_1)} x_\alpha &= 1 && \forall\, r\in\mathcal{R} \\
\sum_{\alpha\in\delta^+(\nu)} x_\alpha - \sum_{\alpha\in\delta^-(\nu)} x_\alpha &= 0 && \forall\, \nu\in\mathcal{V}^{\mathcal{R}}_{\mathcal{T}} \setminus \tilde{\mathcal{V}}^{\mathcal{R}}_{\mathcal{T}} \\
x_\alpha &\in \{0,1\} && \forall\, \alpha\in\mathcal{A}^{\mathcal{R}}_{\mathcal{T}}
\end{aligned}$$

periodic timetabling

$$\sum_{\alpha\in\mathcal{A}^{\mathcal{R}}_{\mathcal{T}}(a)} x_\alpha - \sum_{\substack{a'\in A_{\mathcal{T}}\\ a'\sim a}} \sum_{\substack{p\in\mathcal{P}\\ a'\in p}} y_p \geq 0 \quad \forall\, a\in A^{\sim}_{\mathcal{T}}(A), \forall\,(s,t)\in\mathcal{D}$$

coupling

$$\begin{aligned}
\sum_{p\in\mathcal{P}_{st}} y_p &= 1 && \forall\,(s,t)\in\mathcal{D} \\
y_p &\geq 0 && \forall\,(s,t)\in\mathcal{D}, \forall\, p\in\mathcal{P}_{st}.
\end{aligned}$$

passenger routing

− size

+ linear objective
+ different period times
+ differentiated transfer times
+ splittable flow in capacitated case

Time-Expanded Integrated Model

$$
\begin{aligned}
(\mathrm{PTPR}^{\mathcal{T}}) \quad & \min \sum_{(s,t)\in\mathcal{D}} \sum_{p\in\mathcal{P}_{st}} \sum_{a\in p} \tau_a \, y_p && && \text{linear objective} \\
& \sum_{\alpha\in\mathcal{A}^r_{\mathcal{T}}(b^r_1)} x_\alpha = 1 && \forall\, r\in\mathcal{R} \\
& \sum_{\alpha\in\delta^+(\nu)} x_\alpha - \sum_{\alpha\in\delta^-(\nu)} x_\alpha = 0 && \forall\, \nu\in\mathcal{V}^{\mathcal{R}}_{\mathcal{T}} \setminus \tilde{\mathcal{V}}^{\mathcal{R}}_{\mathcal{T}} && \text{periodic timetabling} \\
& x_\alpha \in \{0,1\} && \forall\, \alpha\in\mathcal{A}^{\mathcal{R}}_{\mathcal{T}} \\
& \sum_{\alpha\in\mathcal{A}^{\mathcal{R}}_{\mathcal{T}}(a)} M\, x_\alpha - \sum_{(s,t)\in\mathcal{D}} \sum_{\substack{a'\in A_{\mathcal{T}} \\ a'\sim a}} \sum_{\substack{p\in\mathcal{P}_{st} \\ a'\in p}} y_p \geq 0 && \forall\, a\in A^{\sim}_{\mathcal{T}}(A) && \text{coupling} \\
& \sum_{p\in\mathcal{P}_{st}} y_p = d_{st} && \forall\,(s,t)\in\mathcal{D} && \text{passenger routing} \\
& y_p \geq 0 && \forall\,(s,t)\in\mathcal{D}, \forall\, p\in\mathcal{P}_{st}.
\end{aligned}
$$

– (size)

– weak LP-relaxation

+ linear objective

+ different period times

+ differentiated transfer times

+ splittable flow in capacitated case

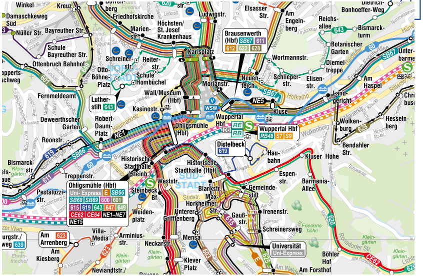
Content

Wuppertal

Wuppertaler Stadtwerke WSW

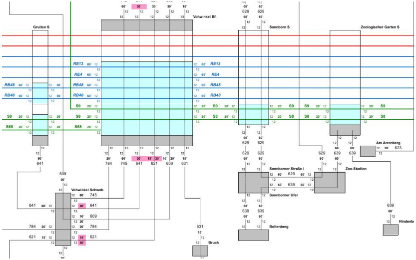
- Wuppertal core network

stations:	158
directed arcs:	460
OD-nodes:	229
OD-pairs:	45 254
lines:	71
period times:	10, 15, 20, 30, or 60 min



- time-expanded networks

86 386	nodes
431 604	passenger arcs
3 990	line arcs



First Computational Results for $(\mathrm{PTPR}^{\mathcal{T}})$

- computations with SCIP version 3.1.0 (Cplex 12.6 as LP-solver)
- column generation algorithm for passenger path-flow variables
- primal heuristic: Route passengers sorted by demand $\rightarrow$ fix timetable
- preprocessing: remove all passengers that do not transfer

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Initial LP: 3 990 binary variables, 76 519 constraints

Solving time root node : 985s

Gap root node : 12.29%

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Comparison to reference timetable WSW 2013

	travel time in min	transfer time in min
WSW 2013	2 630 211.97	171 985.41
WSW*	2 597 571.95	131 456.07
improvement	1.24%	23.57%

Bibliography I

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Bibliography II